Software Model Checking Using Linear Constraints

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Model Checking

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What is the common approach?

Abstract, Check, Refine: the CEGAR approach

Given a program P and a property ϕ to check,

1. Build an abstract model $\hat{P}$ of P

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   - If $\hat{P} \models \phi$, then $P \models \phi$, and return TRUE.

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Note that each abstraction is conservative, that is, it preserves the property to check: $\hat{\phi} = \phi$

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     - If $\hat{C}$ is also a concrete counterexample, then $P \not\models \phi$ and return `FALSE`.

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        - If $\hat{C}$ is spurious, then refine $\hat{P}$ and go back to Step 2.

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How to build the abstract model $\hat{P}$?

Abstraction

Predicate abstraction

Given

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Given

- a concrete program P,

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Given

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- a set of predicates E,

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let α be the abstraction function such that

$$\alpha(P, E) \rightarrow B,$$

where B is a Boolean Program.

Abstraction

Boolean Programs

Imperative programs whose variables are of type boolean only.

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  - iteration constructs,
  - conditional constructs,
  - procedure calls (also with recursion) with call-by-value parameter passing;

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The abstraction function is such that

- it preserves the control-flow graph of the concrete program. Therefore, Boolean Programs support:
  - iteration constructs,
  - conditional constructs,
  - procedure calls (also with recursion) with call-by-value parameter passing;
- each boolean variable b_i is in a one-to-one relation with a concrete predicate $e_i \in E$.

Example Boolean Program

```
decl g;
main()
begin
 decl h;
 h:= !g;
 A(g,h);
 skip;
 A(g,h);
 skip;
 if (g) then skip;
 else skip;
 fi
end
```

```
A(a1,a2)
begin
 if (a1) then
  A(a2,a1);
  skip;
 else
  g:= a2;
 fi
end
```

Issues in Software Model Checking
The CEGAR approach

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Issues in Software Model Checking

The CEGAR approach

- Abstraction and Refinement are 'expensive', as they make intensive use of theorem provers.
- Boolean abstraction is very coarse, and requires many Abstract-Check-Refine loops in order to output a solution.
- The whole Abstract/Check/Refine loop may not terminate, even by choosing an abstraction (like Boolean Programs) for which the model checking procedure is terminating.

Linear Programs

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Linear Programs

Imperative programs such that:

- variables range over a numerical domain $\mathcal{D}$
- expressions are of the form

$$c_0 + c_1 x_1 + \ldots + c_n x_n,$$

where $c_0, \ldots, c_n$ are numeric constants and $x_1, \ldots, x_n$ are program variables ranging over $\mathcal{D}$.

Example Linear Program

```
int x,y;
main(){
 int k;
 k=0; x=10;   y=20;
 while(k<50){
  swap();
  k=k+1;
 }
 if(x==10){
  ERROR:  ;
 } else {
   ;
 }
}
```

```
swap(){
 int c;
 c=x;
 x=y;
 y=c;
}
```

Overview of the procedure

Similarly to the SLAM's Bebop approach (based on the Interprocedural Dataflow Analysis procedure of Reps, Horwitz, Sagiv [POPL, 1995]), the procedure computes reachability of vertices of the control-flow graph, using:

- Path Edges to represent the reachability status of vertices

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No need to compute twice the effects of procedure calls for the same input!

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The procedure works by incrementally storing in a worklist the next statement to be analyzed, by computing Path Edges and Summary Edges accordingly, and by removing the processed statement from the worklist.

Overview of the procedure

Example

```
int x,y;
main(){
 x=0;
 y=0;
 while(x<1){
  x=x+1;
  p(x,y);
 }
 ;
}
p(a,b){
 b=b+a;
}
```

$(x' = x) \wedge (y' = y)$

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    p(x,y);
  }
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}
```

Annotations beside the code:

- `main(){`: $(x' = x) \wedge (y' = y)$
- `x=0;`: $(x' = 0) \wedge (y' = y)$
- `y=0;`: $(x' = 0) \wedge (y' = 0)$
- `while(x<1){`: $(x' = 0) \wedge (y' = 0)$

Overview of the procedure

Example

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int x,y;
main(){
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    x=x+1;
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$(x' = 0) \wedge (y' = 0)$

$(x' = 1) \wedge (x = 0) \wedge (y' = 0)$

Overview of the procedure

Example

```
int x,y;
main(){
  x=0;
  y=0;
  while(x<1){
    x=x+1;
    p(x,y);
  }
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}
p(a,b){
  b=b+a;
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```

- main(){ : $(x' = x) \wedge (y' = y)$
- x=0; : $(x' = 0) \wedge (y' = y)$
- y=0; : $(x' = 0) \wedge (y' = 0)$
- while(x<1){ : $(x' = 0) \wedge (y' = 0)$
- x=x+1; : $(x' = 1) \wedge (x = 0) \wedge (y' = 0)$
- p(a,b){ b=b+a; : $(a = 1) \wedge (a' = a) \wedge (b = 0) \wedge (b' = b)$

Overview of the procedure

Example

```
int x,y;
main(){
  x=0;
  y=0;
  while(x<1){
    x=x+1;
    p(x,y);
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  ;
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p(a,b){
  b=b+a;
}
```

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$(a = 1) \wedge (a' = a) \wedge (b = 0) \wedge (b' = b)$

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 x=0;
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main(){ — $(x' = x) \wedge (y' = y)$

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y=0; — $((x' = 0) \wedge (y' = 0)) \vee ((x' = 1) \wedge (x = 1) \wedge (y' = 1) \wedge (y = 0))$

while(x<1){ — $(x' = 0) \wedge (y' = 0)$

x=x+1; — $(x' = 1) \wedge (x = 0) \wedge (y' = 0)$

b=b+a; — $(x = 1) \wedge (x' = 1) \wedge (y = 0) \wedge (y' = 1)$

Overview of the procedure

Example

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int x,y;
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; : $(x' = 1) \wedge (x = 1) \wedge (y' = 1) \wedge (y = 0)$

b=b+a; : $(x = 1) \wedge (x' = 1) \wedge (y = 0) \wedge (y' = 1)$

Overview of the procedure

Example 2

```
int x,y;
main(){
 int k;
 k=0; x=10;   y=20;
 while(k<50){
  swap();
  k=k+1;
 }
  ...
}
```

```
swap(){
 int c;
 c=x;
 x=y;
 y=c;
}
```

$$((x' = 10) \wedge (x = 20) \wedge (y' = 20) \wedge (y = 10))$$
$$\vee$$
$$((x' = 20) \wedge (x = 10) \wedge (y' = 10) \wedge (y = 20))$$

Overview of the procedure

Symbolic Representation

We represent Path Edges and Summary Edges by means of Abstract Disjunctive Linear Constraints (ADLCs for short).

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Symbolic Representation

ADLCs

An Abstract Disjunctive Linear Constraint is an expression of the form

$$\lambda \mathbf{x} \lambda \mathbf{x}'. \bigvee_i \bigwedge_j c_{ij}.$$

Overview of the procedure

Symbolic Representation

ADLCs

An Abstract Disjunctive Linear Constraint is an expression of the form

$$\lambda \mathbf{x} \lambda \mathbf{x}'. \bigvee_i \bigwedge_j c_{ij}.$$

Main operators over ADLCs:

- $Join(P, \tau)$: computes the ADLC resulting from the application of the transition relation τ to an ADLC P,
- $\sqsubseteq$: entailment,
- $\exists \mathbf{x}$: quantifier elimination,
- $\sqcap$, $\sqcup$, $^{\sim}$: and, or, not.

Eureka

We have implemented eureka, a fully automatic symbolic model checker for Linear Programs.

- Path Edges and Summary Edges are represented symbolically by ADLCs;
- ADLCs are manipulated with a Fourier-Motzkin Constraint Solver;
- It applies quantifier elimination on ADLCs each time a $Join$ operation is performed,
- Entailment checks are provided by ICS 2.0.

Comparison

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- We chose BLAST, the Berkeley Lazy Abstraction Software verification Tool, for a comparison.
- We carried out some quantitative analysis between BLAST and Eureka on our benchmark library of Linear Programs.

The Eureka library of Benchmarks

- Aim: assess the scalability of the model checkers;
- Programs are parametric in a positive integer N (the higher N, the more difficult the program)

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- Programs are parametric in a positive integer N (the higher N, the more difficult the program)

Program name	data	control	iteration	recursion
swap_seq.c(N)	$\surd$			
swap_iter.c(N)	$\surd$	$\surd$	$\surd$	
parity.c(N)	$\surd$	$\surd$		$\surd$
delay_iter.c(N)	$\surd$	$\surd$	$\surd$	
delay_recur.c(N)	$\surd$	$\surd$	$\surd$	$\surd$
sum.c(N)	$\surd$	$\surd$	$\surd$	

Experimental Results

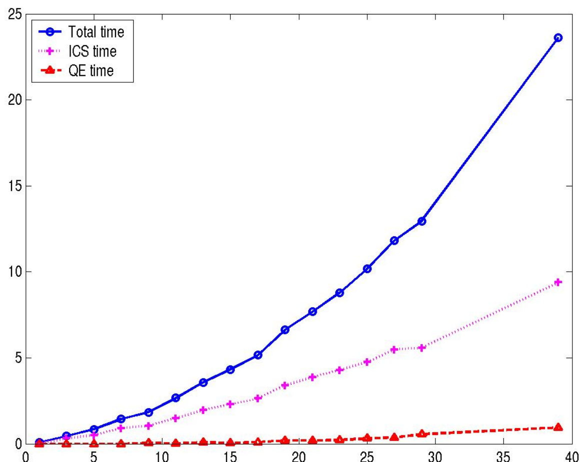
Delay_recur.c(N)

```
main(){
 int x;
 x = 0;
 while(x < N){
  delay(x);
  x = x + 1;
 }
 if (x > 0){
  ;
 } else {
  ERROR:    ;
 }
}
```

```
delay(n){
  int i;
  if (n==0){
    ;
  } else {
    i = n - 1;
    delay(i);
  }
}
```

Experimental Results

Delay_recur.c(N)



Experimental Results

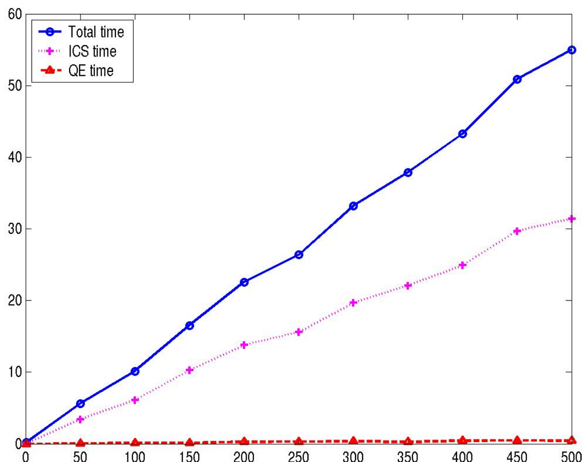
Swap_seq.c(N)

```
int x,y;
main(){
 x=10; y=20;
 swap();
 swap();
 /* 2N times */
 swap();
 swap();
 if (x==10){
   ;
 } else { ERROR: ; }
}
```

```
swap(){
 int c;
 c=x;
 x=y;
 y=c;
}
```

Experimental Results

Swap_seq.c(N)



Experimental Results

Comparison with BLAST

Numbers in table: the hardest instance (N) of the programs that could be verified by the tools.

Program name	Eureka	BLAST
swap_seq.c(N)	500	no
swap_iter.c(N)	100	1
parity.c(N)	50	no
delay_iter.c(N)	150	1000
delay_recur.c(N)	39	no
sum.c(N)	3	2

Summary

- We developed a model checking procedure for software, that uses Linear Constraints;
- We devised a prototype implementation of the procedure called eureka.
- We proposed a set of benchmarks, called the eureka library, to assess the scalability property of our tool and to allow a comparison with BLAST.
- We had good experimental results!
- All numbers, papers and talks are available at URL

  `http://www.ai.dist.unige.it/eureka`.

Future Work

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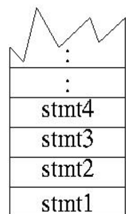
- Complexity Analysis for those classes of programs,
- Extend the library,
- Embed eureka in the Abstraction and Refinement loop (current),
- Do more comparative analysis,
- Use of widening techniques to enforce termination (e.g. Parma Polyhedra Library, Omega tool).

The End

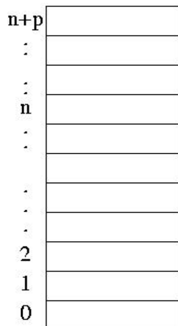
THANK YOU!

Overview of the procedure
Data Structures

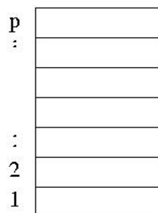
n: # of vertices, p: # of procedures.



$W : \mathbb{N} \rightarrow V_P$
Worklist



$P : \mathbb{N} \rightarrow ADLCs$
Path Edges



$S : \mathbb{N} \rightarrow ADLCs$
Summary Edges

Procedure Transitions

A generic transition of the procedure is of the form

$$(W, P, S) \rightarrow (W', P', S')$$

where the values of W', P', S' depend on the vertex being valuated, that is, on the vertex on top of the worklist.

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Initial state: $(\,First_P(\texttt{main}).\epsilon,\, P, S)$

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A generic transition of the procedure is of the form

$$(W, P, S) \rightarrow (W', P', S')$$

where the values of W', P', S' depend on the vertex being valuated, that is, on the vertex on top of the worklist.
Initial state: $(1.\epsilon, P, S)$, where

- $P(1) = \lambda \mathbf{x}\mathbf{x}'.(\mathbf{x} = \mathbf{x}')$
- $P(i) = \lambda \mathbf{y}\mathbf{y}'.\bot$ for each $0 \leq i \leq (n + p)$, $i \neq 1$
- $S(j) = \lambda \mathbf{y}\mathbf{y}'.\bot$ for each $1 \leq j \leq p$

Procedure Transitions

Conditionals

Let i be a vertex associated with a conditional statement (`while`,`if`,`assert`), then

$$
\begin{aligned}
(W'', P'') &= prop(Fsucc_P(i), W, Join(P(i), \tau_{i,false}), P), \\
(W', P') &= prop(Tsucc_P(i), W'', Join(P(i), \tau_{i,true}), P''), \\
S' &= S.
\end{aligned}
$$

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The purpose of the function $prop$ is to update W and P if 'new' information is provided (checked with the entailment operator).

Procedure Transitions

Procedure Calls

Let i be a vertex associated with statement $s_i = pr(\mathbf{a})$; let $p = Exit_{pr}$, $l_i = SelfLoop(Join(P(i), \tau_i))$, and $r_i = SEJoin(P(i), S(p))$.

Procedure Transitions

Procedure Calls

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$$\begin{aligned} W' &= Insert(sSucc_P(i), is), \\ P' &= P[(P(sSucc_P(i)) \sqcup l_i)/sSucc_P(i)], \\ S' &= S. \end{aligned}$$

Procedure Transitions

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Otherwise,

$$\begin{aligned} (W', P') &= prop(Next_P(i), W, r_i, P), \\ S' &= S. \end{aligned}$$

Procedure Transitions

Exit from Procedure Calls

Let $i \in Exit_P$, $w \in Succ_P(i)$, $c \in Call_P$ such that $w = ReturnPt_P(c)$, and $s = Lift_{\mathbf{c}}(P_i)$.

Procedure Transitions

Exit from Procedure Calls

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Procedure Transitions

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Let $i \in Exit_P$, $w \in Succ_P(i)$, $c \in Call_P$ such that $w = ReturnPt_P(c)$, and $s = Lift_{\mathbf{c}}(P_i)$. If $s \not\sqsubseteq S(i)$ then for each w,

$$
\begin{aligned}
(W', P') &= prop(w, W, SEJoin(P(i), S(i) \sqcup s), P), \\
S' &= S\left[(S(i) \sqcup s)/i\right].
\end{aligned}
$$

Procedure Transitions

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$$\begin{aligned} (W', P') &= prop(w, W, SEJoin(P(i), S(i) \sqcup s), P), \\ S' &= S\left[(S(i) \sqcup s)/i\right]. \end{aligned}$$

Otherwise,

$$\begin{aligned} W' &= W, \\ P' &= P \\ S' &= S. \end{aligned}$$

Procedure Transitions

All other cases

If $i \in V_P \setminus Cond_P \setminus Call_P \setminus Exit_P$, then

$$\begin{aligned} (W', P') &= prop(sSucc_P(i), W, Join(P(i), \tau_i), P), \\ S' &= S. \end{aligned}$$

Procedure Transitions

All other cases

If $i \in V_P \setminus Cond_P \setminus Call_P \setminus Exit_P$, then

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When no more rules are applicable,

- P contains the valuations of all the vertices of the control flow graph,

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- S contains the valuations that show the behaviour of every procedure.

Procedure Transitions

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If $i \in V_P \setminus Cond_P \setminus Call_P \setminus Exit_P$, then

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- P contains the valuations of all the vertices of the control flow graph,
- S contains the valuations that show the behaviour of every procedure.

Result: if the Path Edges for a vertex i are valuated to $\bot$, then s_i is not reachable.